



Same Volume, Different Box

Mission: The factory ships 24 blocks per box, but designers argue about the box shape. Find EVERY box that holds exactly 24 unit cubes and recommend the best design.

Big question: How many different rectangular prisms have the same volume?

★ Warm-up: Factor hunt

List as many as you can.

Factor pairs of 24 ✓ **1×24, 2×12, 3×8, 4×6.**

Is $2 \times 3 \times 4$ the same box as $4 \times 3 \times 2$? ✓ **Yes, just turned.**

Key vocabulary

Factor	A number that multiplies to make another number.
Dimensions	The length, width, and height of a box.
Systematic	Working in an organized order so you don't miss any.
Cube	A box with all edges the same length.

1 Box 1

Find them all

In the simulator: Find and save EVERY box design with volume 24.

How many different box designs have volume 24?

Answer: _____ designs ✓ **6**

2 Box 2

Systematic list

Put the designs in a systematic order (smallest first dimension, then second).

Put in order: $1 \times 1 \times 24 \cdot 1 \times 2 \times 12 \cdot 1 \times 3 \times 8 \cdot 1 \times 4 \times 6 \cdot 2 \times 2 \times 6 \cdot 2 \times 3 \times 4$

1. **$1 \times 1 \times 24$**

2. **$1 \times 2 \times 12$**

3. **$1 \times 3 \times 8$**

4. **$1 \times 4 \times 6$**

5. **$2 \times 2 \times 6$**

6. **$2 \times 3 \times 4$**

3 Box 3

Cube check

Why can't any box with volume 24 be a cube?

☐ No whole number $\times$ itself $\times$ itself = 24 ☐ Cubes are too small ☐ Cubes can only hold 27

✓ **No whole number $\times$ itself $\times$ itself = 24**

4 Box 4

Recommend a design

Which design uses the least cardboard (is the most compact)?

☐ $2 \times 3 \times 4$ ☐ $1 \times 1 \times 24$ ☐ $1 \times 2 \times 12$

✓ $2 \times 3 \times 4$

5 Box 5

Legend: volume 36 Legend only

How many different boxes have volume 36?

Answer: _____ designs ✓ 8

Show your thinking:

6 Box 6

Design memo

Explain how you know you found ALL the boxes with volume 24.

Look for: Describes a systematic method; Uses factors

Exit ticket: answer the big question in your own words

How many different rectangular prisms have the same volume?

Look for: Different boxes can have the same volume. Any three whole numbers whose product is 24 make a box with volume 24: $1 \times 1 \times 24$, $1 \times 2 \times 12$, $1 \times 3 \times 8$, $1 \times 4 \times 6$, $2 \times 2 \times 6$, and $2 \times 3 \times 4$. Rotating a box does not make a new one.

Completion codes

Students see a code like **ABC-I3** when they finish. The first part must match below. The letter is the level (E = Explorer, I = Investigator, L = Legend), and the number at the end is how many stars they earned.

Level	Code starts with
 Explorer	1A6-E
 Investigator	1KD-I
 Legend	1Y2-L

Watch for these misconceptions

- "Volume and area are the same." Area is 2-D square units; volume is 3-D cubic units.
- "Add the dimensions." Volume multiplies them.
- "Composite figures: multiply all the numbers you see." Split into separate prisms first.